Solutions to JEE Main Home Practice Test - 4 | JEE - 2024

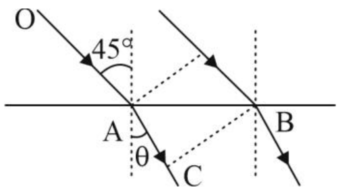
PHYSICS

SECTION-1

1.(D) $AB = \sqrt{2}$ cm

From refraction at air-water boundary,

$$(1)\left(\frac{1}{\sqrt{2}}\right) = (\sqrt{2})\sin\theta \text{ or } \theta = 30°$$



$$\therefore \quad BC = \frac{\sqrt{3}}{\sqrt{2}} \text{ cm}$$

2.(B) $B.E_1 = 7 \times 5.6$

$B.E_2 = 4 \times 7.06 \times 2$

Energy released $= B.E_2 - B.E_1$

$= 17.28 \qquad \approx 17.3$

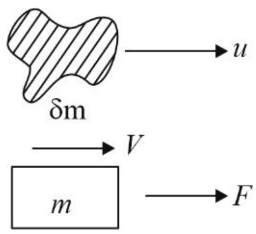
3.(C) A has greater kinetic energy than *P*.

Since the magnetic force acting is zero, *A* and *P* do not move in circular paths.

Also, $E = \dfrac{P^2}{2m}$

and mass of α-particle is less and hence it has greater kinetic energy.

4.(A) $F = m\dfrac{dv}{dt} + (V-u)\dfrac{dm}{dt}$



Here $u =$ velocity of sand $= O$

$m = m_0 + \mu t =$ mass at time t

and $\dfrac{dm}{dt} = \mu$

$$\therefore \quad F(M_0 + \mu t)\frac{dm}{dt} + v\mu$$

$$(F - \mu v)dt = (M_0 + \mu t)dv$$

$$\int_0^t \frac{dt}{M_0 + \mu t} = \int_0^v \frac{dv}{F - \mu v}; \qquad \frac{1}{\mu}[\log(M_0 + \mu t)]_0^t = -\frac{1}{\mu}[\log(F - \mu v)]_0^v$$

$$\log\frac{(M_0}{\ } \ \log\frac{(M_0 + \mu t)}{M_0} = \log\left(\frac{F}{F - \mu v}\right)$$

$$F - \mu v = \frac{M_0 F}{M_0 + \mu t} \quad \Rightarrow \quad v = \frac{Ft}{M_0 + \mu t}$$

5.(B) $\dfrac{1}{2}mv_{max}^2 = 2\,\text{J}$; $v_{max} = 2\,\text{m/s} = A\omega$,

$$v_{avg} = \frac{2A}{T/2} = \frac{4A}{T} = \frac{4A\omega}{2\pi} = \frac{4}{\pi}\text{m/s}$$

6.(D) $n = 2 \longrightarrow 3$

$$47.2eV = (13.6eV)Z^2\left(\frac{1}{4}-\frac{1}{9}\right)$$

$\Rightarrow \quad Z = 5 \quad \boxed{Z=5}$

7.(A) $mL_{ice} = \dfrac{KA(\Delta\theta)t}{d} \Rightarrow \dfrac{4}{3}\pi R^3\rho \times L_{ice} = \dfrac{K4\pi R^2(\Delta\theta)t}{d}$

$\therefore \quad \dfrac{t_1}{t_2} = \dfrac{R_1}{R_2}\times\dfrac{K_2}{K_1}$

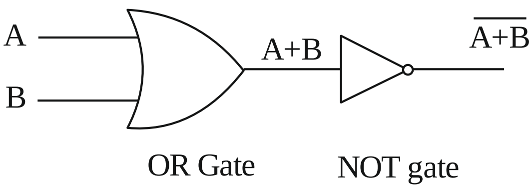
8.(B) Instantaneous Axis of Rotation passes through the point whose velocity is zero
And points at distance less than distance of C.M. from I.O.R. will have velocity.

9.(C) Valid for photon not for electron

10.(C) $fr_{\lim} \geq m\sqrt{a^2 + \left(\dfrac{(at)^2}{R}\right)^2}$

$\mu mg \geq m\sqrt{a^2 + \left(\dfrac{a^2t^2}{R}\right)^2}$; $\qquad \mu g \geq \sqrt{a^2 + \left(\dfrac{a^2t^2}{R}\right)^2}$

11.(B)



A	B	output
0	0	1
1	0	0
0	1	0
1	1	0

NAND Gate

12.(D) after t_1

$d = -\dfrac{1}{2}at_1^2$

$u = at_1$

$S = ut + \dfrac{1}{2}at^2; \quad -\dfrac{1}{2}at_1^2 = at_1t - \dfrac{1}{2}at^2$

$t^2 - t_1t - t_1^2 = 0; \qquad t = (\sqrt{2}+1)t_1$

total time $= (\sqrt{2}+2)t_1$

13.(D) $\rho gh\ \pi r^2 = 2\pi rS\cos\theta \qquad \Rightarrow \qquad r = \dfrac{2S\cos\theta}{\rho gh} = \dfrac{2\times 0.1\times 0.5}{10^3\times 10\times 10} = 10^{-6}\text{m}$

14.(B) $P_0 = B.K.S_0 = B\left(\frac{2\pi}{\lambda}\right)S_0 \quad \Rightarrow \quad P_0 \propto \frac{1}{\lambda}$

Thus, pressure amplitude is highest for minimum wavelength, other parameters B and S_0 being same for all. From given graphs. $\lambda_3 < \lambda_2 < \lambda_1$

15.(D) $\varepsilon_r = \frac{c^2}{v^2\mu_r} = 2.25$

16.(C) $I = i_1\cos(\omega t) + i_2\sin(\omega t)$

$I_{rms}^2 = \left(\frac{i_1}{\sqrt{2}}\right)^2 + \left(\frac{i_2}{\sqrt{2}}\right)^2; \qquad I_{rms} = \frac{1}{\sqrt{2}}(i_1^2 + i_2^2)^{1/2}$

17.(C) $i = neAv_d$ **;** $\qquad v_d = \frac{i}{neA} = \frac{20}{10^{29} \times 1.6 \times 10^{-19} \times 10^{-6}} = 1.25 \times 10^{-3}\,\text{m/sec}$

18.(A) The value of current is given by :

$i = i_0(1 - e^{-t/\tau}) \qquad \Rightarrow \quad \frac{di}{dt} = \frac{i_0}{\tau}e^{-t/\tau}$

Energy stored in the form of magnetic field energy is:

$$U_B = \frac{1}{2}Li^2$$

Rate of increase of magnetic field energy is:

$R = \frac{dU_B}{dt} = Li\frac{di}{dt} = \frac{Li_0^2}{\tau}(1 - e^{-t/\tau})\ e^{-t/\tau}$

This will be maximum when $\frac{dR}{dt} = 0 \quad \Rightarrow \quad e^{-t/\tau} = 1/2$

Substituting :

$R_{\max} = \frac{Li_0^2}{\tau} = \frac{Li_0^2}{\tau}\left(1 - \frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{Li_0^2}{4\tau} = \left[\frac{L(E/R)^2}{4(L/R)}\right] = \frac{E^2}{4R}$

19.(A) Least count = 0.01
zero error = 0.05 cm
$d = (MSR) + (VSR \times LC) + NZE$
$= 2.5\text{ cm} + (6 \times 0.01)\text{ cm} + 0.05\text{ cm} = 2.51\text{ cm}$

20.(C) Cyclic process $\Delta U = 0$
$\Delta Q = \Delta W$
$Q_1 + Q_2 + Q_3 + Q_4 = W_1 + W_2 + W_3 + W_4$
$1040 = 275 + W_4$

SECTION – 2

1.(17) source $I = \frac{P}{A} = \frac{1}{4\pi(0.5)^2}$

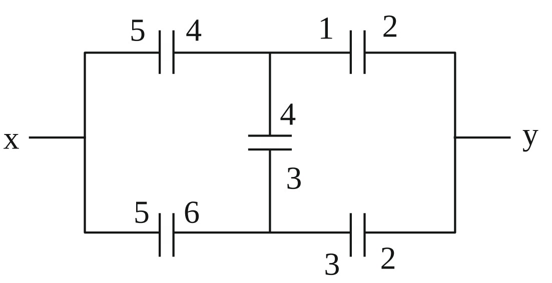
$$\frac{1}{4\pi(0.5)^2} \times \pi \times (1.3\times10^{-10})^2 \times t = 1.8\times1.6\times10^{-19}\,; \qquad t = 17.04$$

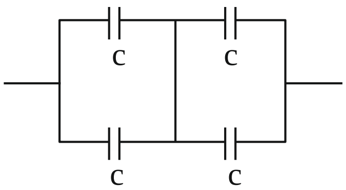
2.(3) $d = \sqrt{2h_T R} + \sqrt{2h_2 R}$

$= \sqrt{2\times36\times6.4\times10^6} + \sqrt{2\times49\times6.4\times10^6}$ = 46.5 km; $\quad x = 3$

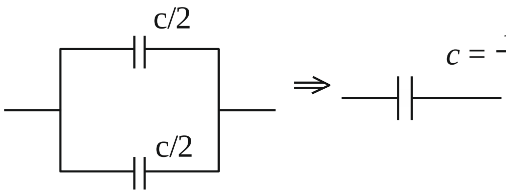
3.(2) $V_c = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R}\left(\rho\frac{4}{3}\pi R^3\right)}$

4.(1)





$c = \frac{A\varepsilon_0}{d}$



$c = \frac{A\varepsilon_0}{d}$

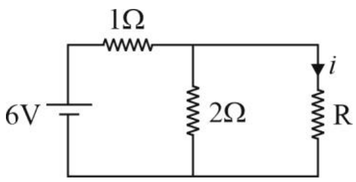
5.(20) $kx = mg\sin\theta$

$$\frac{(2.5)(10)(\sin 30°)}{x} = 1000$$

$$\omega = \sqrt{\frac{k}{m}} = \frac{20\text{rad}}{S}$$

6.(52) $a = \frac{F}{13} \le \mu g\,; \qquad F = (0.4)(10)(13) = 52N$

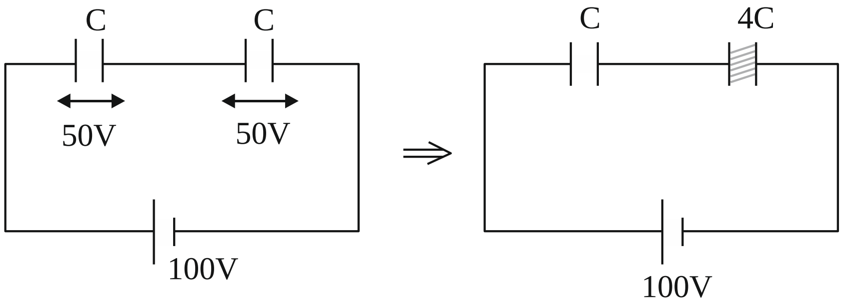
7.(15)



$R = 1 + \frac{2R}{2+R}\,; \qquad R^2 - R - 2 = 0$

$\boxed{R = 2\Omega}$; $\qquad i = 1.5A$

8.(80)



New capacitance = KC, $C' = 4C$

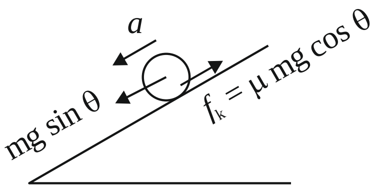
in series $q_1 = q_2$

$C_1V_1 = C_2V_2$; $\frac{V_1}{V_2} = \frac{4C}{C}$

$\frac{V_2}{V_2} = 4$; $V_1 + V_2 = 100$

$\boxed{V_1 = 80V}$

9.(1)



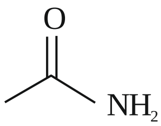
Acceleration and displacement is same in both case hence time is same.

10.(3) $\theta_1 = 0(2) + \frac{1}{2}\alpha(2)^2 = 2\alpha$

$\theta_2 = \frac{1}{2}\alpha(4)^2 - \frac{1}{2}\alpha(2)^2 = 6\alpha$

CHEMISTRY

SECTION - 1

1.(C)  $\xrightarrow{NaOH+Br_2} CH_3-NH_2$ (Hofmann's bromamide Rxn.)

2.(B) HCl itself get oxidise.

3.(C) As the stability of carbocation increases rate of dehydration increases.

4.(D) Acidic strength of different oxides of same element increases with increase in oxidation state of element.

5.(B) For (A), $E^\circ_{cell} = E^\circ_{M/M^+} + E^\circ_{X/X^-} = -0.44 + 0.33V = -0.11V$, i.e., it is negative.

Hence, this reaction is non-spontaneous. For **(B)**, $E^\circ_{cell} = +0.11V$. Hence, it is spontaneous.

6.(A) The most electronegative element is F and next to F is O.

7.(D) Hemiacetal is obtained by reaction between aldehyde and alcohol.

8.(D) Because of sp^3 carbon it is not aromatic.

9.(C) Nitrogen and sulphur is responsible for blood red colouration.

10.(A) H_2O can form hydrogen bonding

11.(B) Bond angles of ClF_3, PF_3, NF_3 and BF_3 are (approx. 180°, approx. 90°), (101°), (106°) and 120°) respectively.

12.(B) The configuration are:

$Zn^+ : [Ar]3d^{10}, 4s^1$

$Fe^{2+} : [Ar]3d^6$

$Ni^{2+} = [Ar]3d^8$; $Cu^+ [Ar]3d^{10}$

13.(C) Lanthanides forms LnC_2, Ln_2C_3, Ln_3C type carbides.

14.(C)

15.(D) Allylic and benzylic positions are highly reactive for S_N1 and S_N2 reaction.

16.(B) Zr(40) and Hf(72) are in the same group. Because of lanthanide contraction they have nearly same size.

17.(A)

18.(A) $Ph-NH_2 \xrightarrow{CHCl_3+\overset{\Theta}{O}H} Ph-\overset{\oplus}{N}\equiv\overset{\Theta}{C} \xrightarrow[H_2O]{LiAlH_4} Ph-\underset{\substack{|\\H}}{N}-CH_3$

19.(D) $[H^+] = \sqrt{K_aC}$

$\sqrt{K_1 \times C_1} = \sqrt{K_2 \times C_2}$

$\sqrt{\frac{C_1}{C_2}} = \sqrt{\frac{K_2}{K_1}} = \sqrt{\frac{1}{4}}$ $\quad\therefore\quad$ $\frac{C_1}{C_2} = \frac{1}{4} = 0.25$

20.(D) ClF_3 has sp^3d-hybridization with two lone pair of electron on Cl.

SECTION - 2

1.(8) $A = \dfrac{E_{1,2}}{2E_{2,1}} = \dfrac{-13.6 \times 2^2 \times 2^2}{2 \times 1^2 \times (-13.6) \times 1^2} = \dfrac{16}{2} = 8$

2.(9) $M = \dfrac{\%\text{ by weight} \times 10 \times d}{Mw_2} = \dfrac{30 \times 10 \times 1.095}{36.5} = 9M$

3.(2) $\Delta_r H = n[\Delta H°_{(C=C)}] - (2n+1)\,\Delta H°_{(C-C)}$

as $n >> 1$, then $(2n+1) \approx 2n$

$\Delta_r H = n[590] - 2n[331] = -72n$

(Per mole of propylene); $\Delta_r H° = -72\,kJ/mole$

$\Rightarrow \quad x = 72$ and $\quad \dfrac{x}{36} = 2$

4.(6) $2NH_3(g) \rightleftharpoons N_2(g) + 3H_2(g)$

Rate $= K[NH_3]^0$

Rate = 2

Rate $= \dfrac{1}{3} \times \dfrac{\Delta[H_2]}{\Delta t} = 2 \quad \Rightarrow \quad \dfrac{\Delta[H_2]}{\Delta t} = 6$

5.(4) $P = P°_A + x_B(P°_B - P°_A)$

$99 = 100 + x_b(-20) \Rightarrow x_B = \dfrac{1}{20} \qquad \therefore y_B = \dfrac{80 \times \frac{1}{20}}{99} = \dfrac{4}{99}$

% mole of B in vapour phase $= \dfrac{4}{99} \times 100 = 4$

6.(1) $Cl_2 + 2e^- \longrightarrow 2Cl^-$

$Cl_2 \longrightarrow 2ClO_3^- + 10e^-$

$5Cl_2 + Cl_2 \longrightarrow 10Cl^- + 2ClO_3^-$

$6Cl_2 \longrightarrow 10Cl^- + 2ClO_3^-$

$12OH^- + 6Cl_2 \longrightarrow 10Cl^- + 2ClO_3^- + 6H_2O$

$3Cl_2 + 6OH^- \longrightarrow ClO_3^- + 5Cl^- + 3H_2O$

$\dfrac{a}{e} = \dfrac{3}{3} = 1$

7.(0) Secondary valency of $PtCl_2.2NH_3$ is four, so no chloride ions are present.

8.(1) milli eq. of H_2O_2 = milli eq. of $KMnO_4$

$(28\times M\times 2)=(10\times 0.1\times 5)$

$M=0.089$

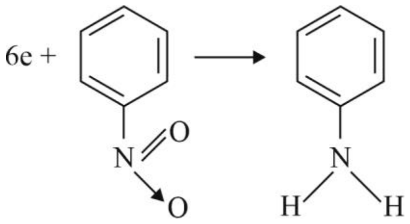
Volume strength $=M\times 11.2=1$

9.(5) $AgCl(s) \rightleftharpoons Ag^{+}(aq)+Cl^{-}(aq)$

$\qquad\qquad\qquad 0.2M \qquad s$

$K_{sp}=[Ag^{+}][Cl^{-}]; \qquad s=\dfrac{10^{-10}}{0.2}=5\times 10^{-10} \quad \Rightarrow \quad y=5$

10.(6)



Required charge for reduction of 1 mole $C_6H_5NO_2=6F$

MATHEMATICS

SECTION-1

1.(C) Given vectors, $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and c, such that $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$

$\Rightarrow \quad \vec{b} \times (\vec{c} - \vec{a}) = 0 \Rightarrow \vec{b} \parallel \vec{c} - \vec{a}$

$\Rightarrow \quad \vec{c} = \vec{a} + \lambda\vec{b} \qquad \Rightarrow \quad \vec{c} - \vec{a} = \lambda\vec{b}$

$\Rightarrow \quad \vec{c} = (1+\lambda)\hat{i} - (2+\lambda)\hat{j} + (1+\lambda)\hat{k}$

$\because \quad \vec{c}.a = 0$ (given)

$\Rightarrow \quad (1+\lambda) + 2(2+\lambda) + (1+\lambda) = 0$

$\Rightarrow \quad 4\lambda + 6 = 0 \qquad \Rightarrow \quad \lambda = -\frac{3}{2}$

$\therefore \quad \vec{c} = -\frac{1}{2}\hat{i} - \frac{1}{2}\hat{j} - \frac{1}{2}\hat{k}$ So $\vec{c}.\vec{b} = -\frac{1}{2} + \frac{1}{2} - \frac{1}{2} = -\frac{1}{2}$

2.(D) Sum, 7: (2, 5), (5, 2); 6: (1, 5), (3, 3), (5, 1)
5: (2,3), (3, 2) ; 4 : (1, 3), (2, 2), (3, 1)
3 : (1, 2), (2, 1) ; 2 : (1, 1)

$P(E) = \frac{13}{36}$

3.(A) Condition for circles touches each other externally and internally

$|C_1C_2| = |r_1 \pm r_2|$

$\sqrt{a^2+b^2} = \left|\left(\sqrt{a^2-c^2}\right) \pm \sqrt{(b^2-c^2)}\right|$

solving we get, $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

4.(A) $A^2 = \begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}\begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$

$A^2 = \begin{pmatrix} \sin^2\alpha & 0 \\ 0 & \sin^2\alpha \end{pmatrix} \qquad = A^2 - I = \begin{pmatrix} \sin^2\alpha - 1 & 0 \\ 0 & \sin^2\alpha - 1 \end{pmatrix}$

$\det\left(A^2 - I\right) = \left(\sin^2\alpha - 1\right)^2 = 0 \qquad \Rightarrow \quad \sin^2\alpha - 1 = 0;\ \alpha = \frac{\pi}{2}$

5.(D) $(x-1)(x-3) = [x](x-1)$

$x = 1$ are solutions

or $x - 3 = [x]$

But $[x] \in (x-1,\ x]$

So, $[x] \neq x - 3$

So only 1 solution

6.(C) It is given that a group of students comprises of 5 body and n girls. The number of ways, in which a team of 3 students can be selected from this group such that each team consists of at least one boy and at least one girls, is = (number of ways selecting one boy and 2 girls) + (number of ways selecting two boys and 1 girl)

$= ({}^5C_1 \times {}^nC_2) + ({}^5C_2 \times {}^nC_1) = 1750$ (given)

$= \left(5 \times \frac{n(n-1)}{2}\right) + \left(\frac{5\times 4}{2} \times n\right) = 1750$

$\Rightarrow \quad n(n-1) + 4n = \frac{2}{5} \times 1750 \Rightarrow n^2 + 3n = 2 \times 350$

$\Rightarrow \quad n^2 + 3n - 700 = 0 \Rightarrow n^2 + 28n - 25n - 700 = 0$

$\Rightarrow \quad n(n+28) - 25(n+28) = 0 \Rightarrow (n+28)(n-25) = 0$

$\Rightarrow \quad n = 25$

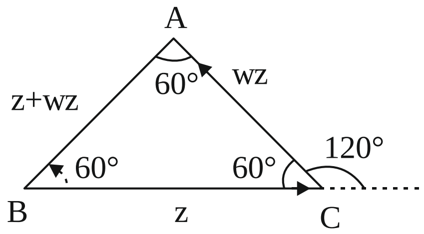
7.(B) $S_{16} - S_5 = T_6 > 1$

$S_7 - S_6 = T_7 < \frac{1}{2}$

$T_6 = \frac{500}{m^2}$ and $T_7 = \frac{500}{m^3}$

$\Rightarrow \quad m^2 < 500$ and $m^3 > 1000 \quad \Rightarrow m \in \{11, 12, ...22\}$

8.(C) Here $|AB| = |BC| = |AC|$



required area = $\frac{\sqrt{3}}{4}|z|^2$

9.(D) Put $z = x + iy$ we get $(x-2)^2 + (y-3)^2 - (x-3)^2 - (y-4)^2 = 2$

$\Rightarrow \quad 2x + 2t = 14 \quad \Rightarrow \frac{x}{7} + \frac{y}{7} = 1$

10.(A) We have $f(x) = \frac{\sin^{-1}(1-\{x\})\cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}(1-\{x\})}$

$\therefore \quad \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$

$= \lim_{h\to 0} \frac{\sin^{-1}(1-\{0+h\})\cos^{-1}(1-\{0+h\})}{\sqrt{2\{0+h\}}(1-\{0+h\})}$

$= \lim_{h\to 0} \frac{\sin^{-1}(1-h)\cos^{-1}(1-h)}{\sqrt{2h}(1-h)}$

$= \lim_{h\to 0} \frac{\sin^{-1}(1-h)}{(1-h)} \lim_{h\to 0} \frac{\cos^{-1}(1-h)}{\sqrt{2h}}$

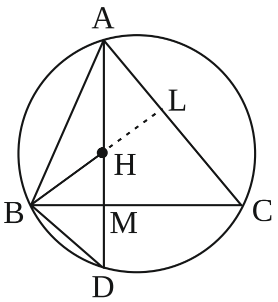
In second limit put $1-h=\cos\theta$

$$=\lim_{h\to 0}\frac{\sin^{-1}(1-h)}{(1-h)}\lim_{\theta\to 0}\frac{\cos^{-1}(\cos\theta)}{\sqrt{2}(1-\cos\theta}$$

$$=\lim_{h\to 0}\frac{\sin^{-1}(1-h)}{(1-h)}\lim_{\theta\to 0}\frac{\theta}{2\sin(\theta/2)}$$

$$=\sin^{-1}1\times 1=\pi/2$$

11.(B)



Let orthocentre be H (5, 8)

Now, $\angle HBM=\frac{\pi}{2}-C$

also, $\angle DBC=\angle DAC=\frac{\pi}{2}-C$

Hence, ΔBMH and ΔBMD are congruent.

$\Rightarrow \quad HM=MD$

$\Rightarrow \quad D$ is image of H in the line $x-y=0$ which is D (8, 5)

Thus, equation of circumcircle is

$$(x-2)^2+(y-3)^2=(8-2)^2+(5-3)^2$$

i.e., $\quad x^2+y^2-4x-6y-27=0$

12.(B) $I=\int_{-\pi}^{\pi}\frac{(-x)\sin(-x)dx}{e^{-x}+1}=\int_{-\pi}^{\pi}\frac{e^x x\sin x}{e^x+1}dx$

adding (i) and (ii)

$$I=\int_0^{\pi}x\sin x\,dx$$

$$I=\int_0^{\pi}(\pi-x)\sin(\pi-x)dx$$

$$2I=\pi\int_0^{\pi}\sin x\ \ dx$$

$\Rightarrow \quad 2I=2\pi; \qquad I=\pi$

13.(B) $m(A)=4,\ m(B)=2$

$m(A\times B)=8$

Required numbers $={}^8C_3+{}^8C_4+...+{}^8C_8$

$=2^8-({}^8C_0+{}^8C_1+{}^8C_2)=256-37=219$

14.(D) $\left|\frac{1-ai}{b+i}\right|=1 \qquad \Rightarrow |1+ai|^2=|b+i|^2$

$(1+ai)(1-ai)=(b+i)(b-i)$

$1+a^2=b^2+1 \quad \Rightarrow a=\pm b$

Since $ab<0, \quad \therefore\ a=-b$ …..(i)

Using

$|z-1|=|2z|$

$\Rightarrow \qquad |z-1|^2=(2|z|)^2$

$\Rightarrow \qquad (z-1)(\bar{z}-1)=4z\bar{z}$

$\Rightarrow \qquad z\bar{z}-z-\bar{z}+1=4z\bar{z}$

$3z\bar{z}+z+\bar{z}-1=0$

$3|z|^2+z+\bar{z}-1=0$ ….(ii)

Since $z = a + ib$ satisfies (i)

$3(a^2+b^2)+(a+ib)+(a-ib)-1=0$

$3(a^2+b^2)+2a-1=0$ ….(iii)

Solving (i) and (iii)

We get $3(a^2+b^2)+2a-1=0$

$3(a^2+b^2)+2a-1=0$

$6a^2+2a-1=0$

$a=\frac{-2\pm\sqrt{28}}{12}=\frac{-2\pm2\sqrt{7}}{12}$

Case-I

$a=\frac{-1+\sqrt{7}}{6}$

$[a]=0$

$b=-a=\frac{1-\sqrt{7}}{6}$

$\therefore \qquad \frac{1+[a]}{4b}=\frac{1+0}{4\left(\frac{1-\sqrt{7}}{6}\right)}=\frac{3}{2(1-\sqrt{7})}$

Case-II

$a=\frac{-1-\sqrt{7}}{6}$

$[a]=-1$

$b=-a=\frac{1+\sqrt{7}}{6}$

$\therefore \qquad \frac{1+[a]}{4b}=\frac{1-1}{4\left(\frac{1+\sqrt{7}}{6}\right)}=0$

As no option matches, therefore it should be Bonus.

15.(B) The given differential equation is

$$\frac{2+\sin x}{y+1}\frac{dy}{dx}=-\cos x, \qquad \Rightarrow \frac{dy}{y+1}=-\frac{\cos x}{2+\sin x}dx$$

Integrating both sides

$$\int \frac{dy}{y+1}=\int \frac{(-\cos x)dx}{2+\sin x}$$

In $|y+1|=-\ln|2+\sin x|+\ln C$

$\Rightarrow \quad \ln|y+1|+\ln|2+\sin x|=\ln C$

$\Rightarrow \quad \ln|(y+1)(2+\sin x)|=\ln C$

$\because \quad y(0)=1 \Rightarrow \ln 4=\ln C \Rightarrow C=4 \qquad \therefore \qquad (y+1)(2+\sin x)=4$

$$y\left(\frac{\pi}{2}\right)=\frac{1}{3}$$

16.(C) $T_3=T_{2+1}={}^5C_2\,x^3\times\left(x^{\log_{10}^x}\right)^2=10^6$

$\Rightarrow \quad (x)^{3+2\log_{10}^x}=10^5$

$x^{3+2\log_{10}x}=10^5$

$\Rightarrow \quad (3+2\log_{10}x)(\log_{10}x)=5$

put, $\log_{10}^x=t$

$\Rightarrow \quad (3+2t)t=5$

solving $t=1,-5/2$

$\log_{10}^x=1 \quad or \quad \log_{10}^x=-5/2$

$\Rightarrow \quad x=10 \qquad \Rightarrow x=10^{-5/2}$

17.(C) We have $\tan^{-1}\left(x+\frac{3}{x}\right)-\tan^{-1}\left(x-\frac{3}{x}\right)=\tan^{-1}\frac{6}{x}$

$$\Rightarrow \quad \tan^{-1}\left(\frac{\left(x+\frac{3}{x}\right)-\left(x-\frac{3}{x}\right)}{1+\left(x+\frac{3}{x}\right)\left(x-\frac{3}{x}\right)}\right)=\tan^{-1}\left(\frac{6}{x}\right) \qquad \Rightarrow \qquad x^2-\frac{9}{x^2}=0 \Rightarrow x^4=9$$

Hence, $\left(5x^8-4x^4+7\right)=5(81)-4(9)+7=405-36+7=412-36=376$

18.(C) $f(x)=x+\cfrac{1}{2x+\cfrac{1}{2x+\cfrac{1}{2x+....\infty}}}=x+\frac{1}{x+f(x)}$

$\Rightarrow \quad f(x)-x=\dfrac{1}{x+f(x)} \qquad \Rightarrow \qquad f^2(x)-x^2=1$

differentiating w.r.t. x

$\Rightarrow \quad 2f(x).f'(x)-2x=0 \quad \Rightarrow \quad f(x).f'(x)=x \qquad \Rightarrow \qquad f(50).f'(50)=50$

19.(A) $S=\cot^{-1}\left(\frac{9}{2}\right)+\cot^{-1}\left(\frac{33}{4}\right)+\cot^{-1}\left(\frac{129}{8}\right)+\ldots\infty$

$T_1=\cot^{-1}\left(\frac{9}{2}\right)=\tan^{-1}\left(\frac{2}{9}\right)=\tan^{-1}\left(\frac{2}{1+4\times 2}\right)=\tan^{-1}4-\tan^{-1}2$

$T_2=\cot^{-1}\left(\frac{33}{4}\right)=\tan^{-1}\left(\frac{4}{33}\right)=\tan^{-1}\left(\frac{4}{1+8\times 4}\right)=\tan^{-1}8-\tan^{-1}4$

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$T_n=\tan^{-1}2^{n+1}-\tan^{-1}(2)$

$S_n=\Sigma T_n=\tan^{-1}2^{n+1}-\tan^{-1}2$

As $n\to\infty$

$S_\infty=\frac{\pi}{2}-\tan^{-1}2=\cot^{-1}2$

20.(B) Given $y=2^{x(x-1)}$

$\Rightarrow \quad x(x-1)=\log_2 y$

$\Rightarrow \quad x^2-x-\log_2 y=0$ (reject the-ve sign)

$\Rightarrow \quad x=\frac{1\pm\sqrt{1+4\log_2 y}}{2}$

$\Rightarrow \quad f^{-1}(x)=\frac{1}{2}\left[1+\sqrt{1+4\log_2 x}\right]$

SECTION-2

1.(25) $\log_2\left(9^{2\alpha-4}+13\right)-\log_2\left(3^{2\alpha-4}\cdot\frac{5}{2}+1\right)=2$

$\frac{9^{2\alpha-4}+13}{3^{2\alpha-4}\cdot\frac{5}{2}+1}=4$

Let $3^{2\alpha-4}=t$

$t^2+13=10t+4$

$t^2-10t+9=0$

$\therefore t=9, 1$

$\Rightarrow \quad \alpha=3,\ 2 \qquad \Rightarrow \sum\alpha=3+2=5$ *and* $\sum(\alpha+1)^2=(3+1)^2+(2+1)^2=25$

Roots of $x^2-50x+25\beta=0$ are real

$\Rightarrow \quad D\le 0 \ \Rightarrow 2500-100\beta\ge 0 \quad \Rightarrow \beta\le 25 \quad \Rightarrow \beta_{\max}=25$

2.(1) We have $2^4=16=17-1$ $\qquad\therefore\qquad$ $2^{2000}=\left(2^4\right)^{500}=(17-1)^{500}$

$={}^{500}C_0 17^{500}-{}^{500}C_1 17^{499}+{}^{500}C_2 19^{498}+{}^{500}C_3 19^{497}+\ldots-{}^{500}C_{499}(17)+(-1)^{500}$

$=17m+1$, Hence m is some positive integer.

3.(31) $y = \frac{(60-3x)}{4}$

$x = 1,\ y = \frac{57}{4} = 14.25$

$(1, 1)(1, 2)........(1, 14) \Rightarrow 14\ pts$

If $x = 2,\ y = \frac{27}{2} = 13.5$

$(2, 2)(2, 4).......(2, 12) \qquad \Rightarrow 6\ pts$

If $x = 3,\ y = \frac{51}{4} = 12.75$

$(3, 3)\ (3, 6).......(3, 12) \qquad \Rightarrow 4\ pts$

If $x = 4,\ y = 12$

$(4, 4)\ (4, 8) \quad \Rightarrow 2\ pts$

If $x = 5,\ y = \frac{45}{4} = 11.25$

$(5,5)\ (5, 10) \quad \Rightarrow 2\ pts$

If $x = 6,\ y = \frac{21}{2} = 10.5$

$(6, 6) \quad \Rightarrow 1\ pt$

If $x = 7,\ y = \frac{39}{4} = 9.75$

$(7, 7) \Rightarrow 1\ pt$

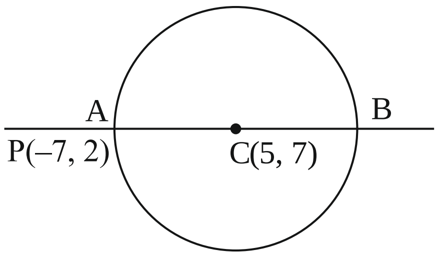
If $x = 8,\ y = 9$

$(8, 8) \Rightarrow 1\ pt$

If $x = 9,\ y = \frac{33}{4} = 8.25 \quad \Rightarrow \text{no. pt}$

Total = 31 *pts*.

4.(2) Centre of the circle is C(5, 7), Let P(–7, 2) be the given point and the line joining P and C meet the circle at A and B



Now, $PC = \sqrt{(12)^2 + (5)^2} = 13$

radius of the circle is $\sqrt{5^2 + 7^2 + 51} = 5\sqrt{5}$

$p = PB = 13 + 5\sqrt{5},\ q = PA = 13 - 5\sqrt{5}$

G.M. of $p, q = \sqrt{pq} = \sqrt{(13)^2 - \left(5\sqrt{5}\right)^2} = 2\sqrt{11} = s$

5.(32) Number divisible by 3 = 18 cases

= 1,2,3,– 6 cases $= 2,3,4-6$ cases $=1,3,5-6$ cases $= 3,4,5-6$ cases

Numbers divisible by 5 = 4 × 3 × 1 = 12

Nos divisible by both = 4

Numbers divisible by 30 or 5 = 24 + 12 – 4 = 32

6.(5) Given sample space (S) of all 3 × 3 matrices with entries from the set {0, 1} and events

$E_1 = \{A \in S : \det A = 0\}$ and

$E_2 = \{A \in S : \text{sum of entries of } A \text{ is } 7\}$

For event E_2, means sum of entries of matrix A is 7.

then we need seven 1s and two 0s.

$\therefore$ Number of different possible matrices = $\dfrac{9!}{7!2!}$ $\Rightarrow$ $n(E_2) = 36$

For event $E_1, |A| = 0,$ both the zeroes must be in same

$\therefore$ Number of matrices such that their determinant is zero

$$= 6 \times \frac{3!}{2!} = 18 = n(E_1 \cap E_2)$$

$\therefore$ Required probability, $P\left(\dfrac{E_1}{E_2}\right) = \dfrac{n(E_1 \cap E_2)}{n(E_2)} = \dfrac{18}{36} = \dfrac{1}{2} = 0.50$

7.(1) $\dfrac{x^2 - y^2}{x^2 + y^2} = \sin\{\ln a\}$

put $y = x\tan\theta,$ $\cos 2\theta = \sin\{\ln a\}$

$\Rightarrow$ $2\theta = \cos^{-1}\{\sin(\ln a)\}$ $\Rightarrow$ $\theta = \dfrac{1}{2}\cos^{-1}(\sin(\ln a))$

$\tan^{-1}\left(\dfrac{y}{x}\right) = \dfrac{1}{2}\cos^{-1}(\sin(\ln a))$ $\Rightarrow$ $\dfrac{y}{x} = \tan\left(\dfrac{1}{2}\cos^{-1}(\sin(\ln a))\right)$

$\Rightarrow$ $\dfrac{\dfrac{x\,dy}{dx} - y}{x^2} = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x}$

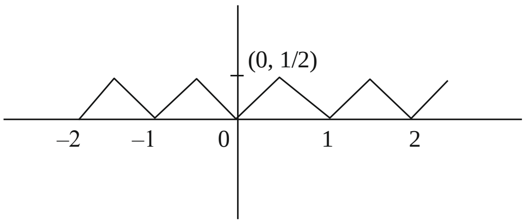
8.(1) As f is continuous so $f(0) = \lim\limits_{x\to 0} f(x) = \lim\limits_{n\to\infty} f(x_n)$

for any sequence x_n such that $\lim\limits_{x\to\infty} x_n = 0.$

Thus $f(0) = \lim\limits_{n\to\infty} f\left(\dfrac{1}{4n}\right) = \lim\limits_{n\to\infty}\left(\left(\sin e^n\right)e^{-n^2} + \dfrac{1}{1+\dfrac{1}{n^2}}\right) = 0 + 1 = 1\left(\lim\limits_{n\to\infty}(\sin e^n\right)e^{-n^2} = 0$

as $\left|\left(\sin e^n\right)e^{-n^2}\right| \le e^{-n^2}$ and $e^{-n^2} \to 0$ as n $\to \infty$)

9.(50) min $\left(\{x\}, \{-x\}\right)$



$\int_{-100}^{100} f(x)dx$ area of 200 triangles shown as solid dark lines in the diagram $= 200 \times \left(\frac{1}{2} \times 1 \times \frac{1}{2}\right) = 50$

10.(6) $p(E_1) = x,\ p(E_2) = y,\ p(E_3) = z$

$\alpha = x(1-y)(1-z)\ \ \beta = (1-x)y(1-z)\ \ y = (1-x)(1-y)z$

Also $p = (1-x)(1-y)(1-z)$

Given $(\alpha - 2\beta)p = \alpha\beta$ *and* $(\beta - 3\gamma)p = 2\beta\gamma$

$(\alpha - 2\beta)(1-x)(1-y)(1-z) = xy(1-x)(1-y)(1-z)$

$(1-z)\left[x(1-y) - 2(1-x)y\right] = xy(1-z)$

$x - xy - 2y + 2xy = xy$

$\boxed{x = 2y}$ $\quad\ldots(1)$

And from 2nd eqn. i.e., $(\beta - 3y)p = 2\beta\gamma$

$(\beta - 3\gamma)(1-x)(1-y)(1-z) = 2(1-x)y(1-z)(1-x)(1-y)$

$= (1-x)\left[y(1-z) - 3(1-y)z\right] = 2(1-x)(yz)$

$y - yz - 3z + 3yz = 2yz$

$\boxed{y = 3z}$ $\quad\ldots(2)$

$\therefore$ From I and II $\dfrac{x}{z} = \dfrac{2y}{y/3} = 6$